Chapter One Justification Handout [How to write a good justification]

Topic One – The Intermediate Value Theorem [IVT]

The IVT is used to prove the existence of some specified y-value on a given domain. If a function ***is continuous on a closed interval***, then we may use IVT to show that the function must pass through all of the y-values between the two endpoints.

If f is continuous on the closed interval $[a, b]$ and k is a number such that $f(a) < k < f(b)$, then there is at least one number c, in $a < c < b$, such that $f(c) = k$. [this is not a curse word]

An example with a function:

Let f be the function given by $f(x) = -e^{x}\cos x$. Show [without using a calculator] that there exists at least one zero on the interval $[0, \pi]$

This function is continuous on the interval $[0, \pi]$ so IVT must apply.

$f(0) = -1$ so, $f(0) < 0$ and $f(\pi) = e^{\pi}$ so, $f(\pi) > 0$

Here is our justification:

Since f(x) is continuous, and f(_) = _
and f(_) = _, by the IVT there exists
a "c", _ ≤ c ≤ _, such that f(c) ___

Here is an example of a typical AP multiple-choice question:

Let f be a continuous function. Selected values of f are given in the table below.

x	1	2	3	5	8
$f(x)$	-2	3	1	-5	7

What is the least number of solutions does the equation $f(x)=\frac{1}{2}$ have on the closed interval [1, 8]?

(A) None

(B) One

(C) Two

(D) Three

(E) Four

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Now let's work on justifying a free response question.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	-10	-15	-10	-7	-5	0	13

A toy car travels on a straight path. During the time interval $0 \le t \le 60$ seconds, the toy car's velocity v, measured in feet per second is a continuous function.

For $0 < t < 60$ must there be a time t when $v(t) = -2$?

An actual AP Free Response Question [well, part of one]

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	−4	4	2
4	−1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

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How to prove that a function is continuous at x=c

What 3 things must be true?

Example:

Let $f(x)$ be the piecewise function described as:

$$f(x) = \begin{cases} x^2 + x + 1 & \text{if } x \leq 2 \\ 11 - 2x & \text{if } x > 2 \end{cases}$$

Is $f(x)$ continuous at $x = 2$?

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Problems:

1. $f(x)$ ***and*** $g(x)$ ***are continuous functions for all*** $x \in$ ***Reals. The table below has values for the functions for selected values of*** x***. The function*** $h(x) = g(f(x)) + 2$

x	$f(x)$	$g(x)$
1	3	4
3	9	-10
5	7	5
7	11	25

Explain why there must be a value c for $1 < c < 5$ such that $h(c) = 0$.

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2.

The functions f and g are continuous. The continuous function h is given by $h(x) = f(g(x)) - x$. The table below gives values of the functions. Explain why there must be a value t for $1 < t < 4$ such that $h(t) = -1$.

x	1	2	3	4
$f(x)$	0	8	-3	6
$g(x)$	3	4	1	2